Sydney Boys' High School

MOORE PARK, SURRY HILLS

2014
November
Year 11
HSC Assessment Task 1

Mathematics

General Instructions

- Reading Time – 5 Minutes.
- Working time – 90 Minutes.
- Write using black or blue pen.
- Pencil may be used for diagrams.
- Board approved calculators maybe used.
- **ALL** necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for untidy or badly arranged work.
- Answer in simplest **EXACT** form unless otherwise instructed.

Total Marks – 90

- Attempt all Questions 1 − 5
- The mark value of each question is shown on the right hand side.
- Each section is to be answered in a **NEW** writing booklet, clearly labelled **Questions 1**, **Question 2** and so on.

Examiner: *J.Millar*

Note: This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Question 1 (21 Marks) - Start a NEW writing booklet.

Marks

(a) Find the equation of the line that intersects the x −axis at $x = -1$ and passes through the point $A(1, -4)$. **1**

(b) Evaluate the $\lim_{x \to 5} \dfrac{2x^2 - 7x - 15}{x - 5}$. **1**

(c) Find the equation of the parabola with focus $(6, -2)$ and directrix the line $x = -4$. **3**

(d) Determine whether the function $f(x) = \dfrac{x^2 + 4}{x^3 - x}$ is odd, even, or neither. **2**

(e) Differentiate with respect to x:

(i) $y = 4x^3 - 2x + \dfrac{1}{x^2} - 1$ **1**

(ii) $y = (2x - 3)\sqrt{x + 2}$ **3**

(iii) $y = \dfrac{4x^2 - 1}{(2x + 1)^2}$ **3**

(f) Find the equation of the line that passes through the point of intersection of the lines $2x + 5y + 19 = 0$ and $4x - 3y - 1 = 0$ and is perpendicular to the line $3x - 2y + 1 = 0$. **3**

(g) A point $P(x, y)$ moves so that its distance from $A(6, 1)$ is twice its distance from $B(-3, 4)$.

(i) Show that the locus of P is a circle. **2**

(ii) Find the centre and radius of the circle. **2**

End of Question 1.

Question 2 (16 Marks) - Start a NEW writing booklet.

Marks

(a) Ketut decides to save money by putting some coins in a jar every day. On the first day he will put in 10c, on the second, 15c, and so on, with the amount he puts into the jar increasing by 5c each day.

(i) How much will he put in on the 15th day? **1**

(ii) For how many days must he save before the jar holds \$37.00? **3**

(b) Given the function $f(x) = \sqrt{10 - x - 3|x - 2|}$. Find the values of x when $f(x) = 2$. **3**

(c) The points $P(2, 3)$, $Q(-11, 8)$, $R(-4, -5)$ are vertices of a parallelogram $PQRS$ where PR is a diagonal.

(i) Find the coordinates of S. **2**

(ii) Hence, determine the area of the parallelogram. **2**

(d) Peter sits for three exams. The probability of him passing Mathematics is 0.65, the probability of him passing PDHPE is 0.85 and the probability of him passing Physics is 0.7. Find the probability that Peter will pass only two subjects. **2**

(e) Find $\dfrac{d}{dx}(3x^2 + 1)$ from first principles. **3**

End of Question 2.

Question 3 (19 Marks) - Start a NEW writing booklet.

Marks

(a) The curve $y = x^3 + ax^2 + bx + 10$ has a stationary point $P(1, 13)$.

(i) Find the value of a and b. **2**

(ii) Find any stationary points of the curve and determine their nature. **3**

(iii) Find the equation of the tangent at the point at which the curve cuts the y −axis. **3**

(iv) Sketch the curve, clearly showing all stationary points. **2**

(b) James borrowed \$40 000 at 1.5% per month reducible interest and he decided to repay the loan plus the interest in equal monthly instalments over 6 years.

(i) Calculate the value of his monthly repayments. **3**

(ii) What annual rate of simple interest is the finance company charging him? **2**

(c) Let α and β be roots of the equation **4**

$$x^2 - px + q = 0.$$

If the equation

$$y^2 - (\alpha + \beta + 1)y + (\alpha^2 + \beta^2) = 0$$

has equal roots, find the value of q in terms of p.

End of Question 3.

Question 4 (14 Marks) - Start a NEW writing booklet.

Marks

(a) State the domain and range of the function $f(x) = 2\sqrt{x-1} + 3$. **2**

(b) The sum of a series is given as $S_n = n^2 - 3^n$.

(i) Find S_{12}. **1**

(ii) Find the 12th term of the series. **2**

(c) Ben and Jerry play a game where they each take turns at throwing two ordinary dice. The winner is the first person to throw a double. Ben throws first.

(i) Show that the probability that Jerry wins the game on his first throw is $\frac{5}{36}$. **1**

(ii) Show that the probability that Jerry wins the game on his first or second throw is given by **1**

$$\frac{5}{36} + \frac{5^3}{6^4}.$$

(iii) Calculate the probability that Jerry wins the game. **3**

(d) The slope at any point $P(x, y)$ of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 2x + k.$$

If the curve touches the x −axis at the point $(2, 0)$:

(i) Find the value of k. **2**

(ii) Find the equation of the curve. **2**

End of Question 4.

Question 5 (20 Marks) - Start a NEW writing booklet.

Marks

(a) At the beginning of every month, starting on the 1st of January 2015, Rhonda plans to deposit $2500 into a superannuation account, paying 6% interest per annum, compounded monthly.

(i) Show that at the end of n months the total amount, A_n, on her superannuation account can be expressed as **3**

$$A_n = 502\ 500(1.005^n - 1).$$

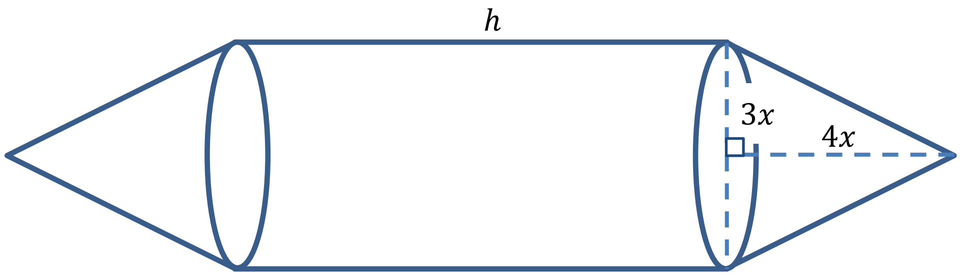
(ii) If Rhonda continues with her superannuation scheme, what would be the amount in her account on the 31st of December 2030? **2**

(iii) Rhonda has estimated that she would be able to retire after the total amount in her superannuation account reaches $600 000. In this case, what would be the date of the first day of her retirement? **3**

(iv) After a serious consideration Rhonda decided that she would retire on the 31st of December 2025 with a total of $600 000, paying larger monthly instalments. Calculate her monthly instalments correct to the nearest cent. **2**

Question 5 continues on the next page....

(b) A man wants to make a tank of capacity $V\ m^3$ from thin metal sheets. The tank is to consist of a right circular cylinder and two identical right circular cones as shown below. The cylinder is of length h metres and radius $3x$ metres. The cone is of radius $3x$ metres and height $4x$ metres.



(i) Express h in terms of x and V. **2**

(ii) The cost per square metre of the metal sheet is $\frac{3k}{2}$. **3**

Let the cost of making the tank be C.

Show that $C = \frac{Vk}{x} + 21\pi k x^2$.

(iii) If $\frac{dC}{dx} = 0$, find x in terms of V. Show that this value of x gives a minimum of C. **3**

(iv) If C is to be a minimum, find the ratio $x : h$. **2**

End of Question 5.

End of Test.

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \qquad n \neq -1;\ x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \qquad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \qquad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \qquad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \qquad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \qquad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \qquad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \qquad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x, \qquad x > 0$

2014 Year 11 HSC Assessment Task 1 - Solutions

Question 1

a $(-1, 0)$ $A(1,-4)$

$m = \frac{0 - -4}{-1 - 1}$

$= \frac{4}{-2}$

$= -2$

$y = -2(x+1)$

$y = -2x - 2$

b

$\lim_{x \to 5} \frac{2x^2 - 7x - 15}{x - 5}$ -30

$\frac{(2x-10)(2x+3)}{2}$

$= (x-5)(2x+3)$

$\lim_{x \to 5} \frac{(x-5)(2x+3)}{(x-5)}$

$\lim_{x \to 5} 2x + 3$

$= 13$

c. $h + a = 6$

$\boxed{k = -2}$

$h - a = -4$

$\therefore 2h = 2$

$\boxed{h = 1}$

$\therefore \boxed{a = 5}$

$(y+2)^2 = 4 \times 5(x-1)$

$y^2 + 4y + 4 = 20x - 20$

$20x - y^2 - 4y - 24 = 0$

d. $f(x) = \frac{x^2 + 4}{x^3 - x}$

$f(-x) = \frac{(-x)^2 + 4}{(-x)^3 - (-x)}$

$= \frac{x^2 + 4}{-x^3 + x}$

$= \frac{x^2 + 4}{-(x^3 - x)}$

$= -\frac{x^2 + 4}{x^3 - x}$

$\therefore f(x)$ is odd

e. i $y = 4x^3 - 2x + x^{-2} - 1$

$y' = 12x^2 - 2 - \frac{2}{x^3}$

ii $y = \underset{u}{(2x-3)}\underset{v}{\sqrt{x+2}}$

$y' = uv' + vu'$

$= (2x-3) \times \frac{1}{2}(x+2)^{-\frac{1}{2}} \times 1 + \sqrt{x+2} \times 2$

$= \frac{1(2x-3)}{2\sqrt{x+2}} + 2\sqrt{x+2}$

iii $y = \dfrac{4x^2-1}{(2x+1)^2}$

$= \dfrac{\cancel{(2x+1)}(2x-1)}{(2x+1)^2}$

$= \dfrac{2x-1}{2x+1}$ $\quad u$
$\quad v$

$u = 2x-1 \qquad v = 2x+1$
$u' = 2 \qquad v' = 2$

$y' = \dfrac{2(2x+1) - 2(2x-1)}{(2x+1)^2}$

$= \dfrac{4x+2-4x+2}{(2x+1)^2}$

$= \dfrac{4}{(2x+1)^2}$

f. $2x + 5y = -19$ ①
$4x - 3y = 1$ ②
① $\times 2$
$4x + 10y = -38$ ③

③ − ②
$13y = -39$
$\boxed{y = -3}$

sub into ①
$2x - 15 = -19$
$2x = -4$
$\boxed{x = -2}$

pt is $(-2, -3)$

$3x - 2y + 1 = 0$
$2y = 3x + 1$
$y = \dfrac{3}{2}x + \dfrac{1}{2}$

$\therefore m_1 = 3/2$

$\therefore m_2 = -2/3$

$(y+3) = -\dfrac{2}{3}(x+2)$
$3y + 9 = -2x - 4$

$2x + 3y + 13 = 0$

g.i $\sqrt{(x-6)^2 + (y-1)^2}$
$= 2\sqrt{(x+3)^2 + (y \mp 4)^2}$

$(x-6)^2 + (y-1)^2 = 4\left[(x+3)^2 + (y \mp 4)^2\right]$

$x^2 - 12x + 36 + y^2 - 2y + 1$
$= 4\left[x^2 + 6x + 9 + y^2 \mp 8y + 16\right]$

$x^2 - 12x + 36 + y^2 - 2y + 1$
$= 4x^2 + 24x + 36 + 4y^2 \mp 32y + 64$

$3x^2 + 36x + 3y^2 - 30y = -63$

$x^2 + 12x + y^2 - 10y = -21$

$(x+6)^2 + (y-5)^2 = -21 + 36 + 25$

$(x+6)^2 + (y-5)^2 = 40$

ii centre $= (-6, 5)$

radius $= \sqrt{40}$

QUESTION 2

1(a)(i) $a=10 \quad d=5$

$T_{15} = 10 + 14\times 5 = 80c$

(ii) $S_n = 3700 = \frac{n}{2}\left(20+5[n-1]\right)$

$7400 = n(15+5n)$

$n^2+3n-1480=0$

$n=-40$ or 37

$n = 37 \quad n>0$

(b) $f(x) = \sqrt{10-x-3|x-2|}$

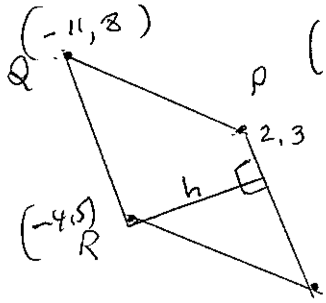
$f(x) = \sqrt{10-x-3x+6}$ or $\sqrt{10-x+3x-6}$

$4 = 16-4x$ or $4 = 4+2x$

$4x=12$ or $2x=0$

$x = 3$ or 0

(c)



(i) $S(-4+13, -5-5)$

$(9, -10)$

(ii) Area $= PS\times h$

$= \sqrt{3-(-10)^2+(2-9)^2} \times \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$

$= \sqrt{139+49} \times \frac{13(-4)+7(-5)-47}{\sqrt{169+49}}$

$= \sqrt{218} \times \frac{134}{\sqrt{218}}$

$A = 134\,u^2$

(d) $0.65 \times 0.85 \times 0.3 +$

$0.65 \times 0.15 \times 0.7 +$

$0.35 \times 0.85 \times 0.7$

$= 0.44225$

or

$\frac{1769}{4000}$

(e) $$\frac{d}{dx}(3x^2+1) = \lim_{h \to 0} \frac{3(x+h)^2+1-(3x^2+1)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2+6xh+3h^2+1-3x^2-1}{h}$$

$$= \lim_{h \to 0} \frac{6xh+3h^2}{h}$$

$$= \lim_{h \to 0} 6x+3h$$

$$= 6x \text{ as } h \to 0$$

3 (a) $y = 1x^3 + ax^2 + bx + 10$

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i) $y' = 3x^2 + 2ax + b$

Data $(1,13)$ on curve $13 = 1 + a + b + 10$
$\Rightarrow a + b = 2$

When $x = 1$, $y' = 0$ $0 = 3 + 2a + b \Rightarrow 2a + b = -3$.

Solve
$$\begin{array}{r} 2a+b=-3 \\ a+b=2 \\ \hline a=-5 \end{array}$$
(1) and $-5 + b = 2$
$b = 7$. (1)

ii)
So $y = 1x^3 - 5x^2 + 7x + 10$
$y' = 3x^2 - 10x + 7$
$y'' = 6x - 10$

When $y' = 0$ $3x^2 - 10x + 7 = 0$
$(3x - 7)(x - 1) = 0$
$x = \frac{7}{3}$, $x = 1$

When $x = 1$, $y = 1 - 5 + 7 + 10 = 13$ $(1, 13)$

When $x = \frac{7}{3}$ $y = \left(\frac{7}{3}\right)^3 - 5\left(\frac{7}{3}\right)^2 + 7\left(\frac{7}{3}\right) + 10 = 11\frac{22}{27}$
or $11.\dot{8}1\dot{4}$

$\left(2\frac{1}{3}, 11\frac{22}{27}\right)$. $1\frac{1}{2}$

When $x = 1$, $y'' = 6 - 10 = -4$ $\Rightarrow (1, 13)$ MAX STAT PT

When $x = 2\frac{1}{3}$ $y'' = 14 - 10 = 4$ $\Rightarrow \left(2\frac{1}{3}, 11\frac{22}{27}\right)$ MIN STAT PT.

$1\frac{1}{2}$

3(a) (iii) curve cuts y axis when $x = 0$

so $y = 1x^3 - 5x^2 + 7x + 10$

$y = 10$

Pt is $(0, 10)$ 1

When $x = 0$ $y' = 3x^2 - 10x + 7$

$m = 7$ 1

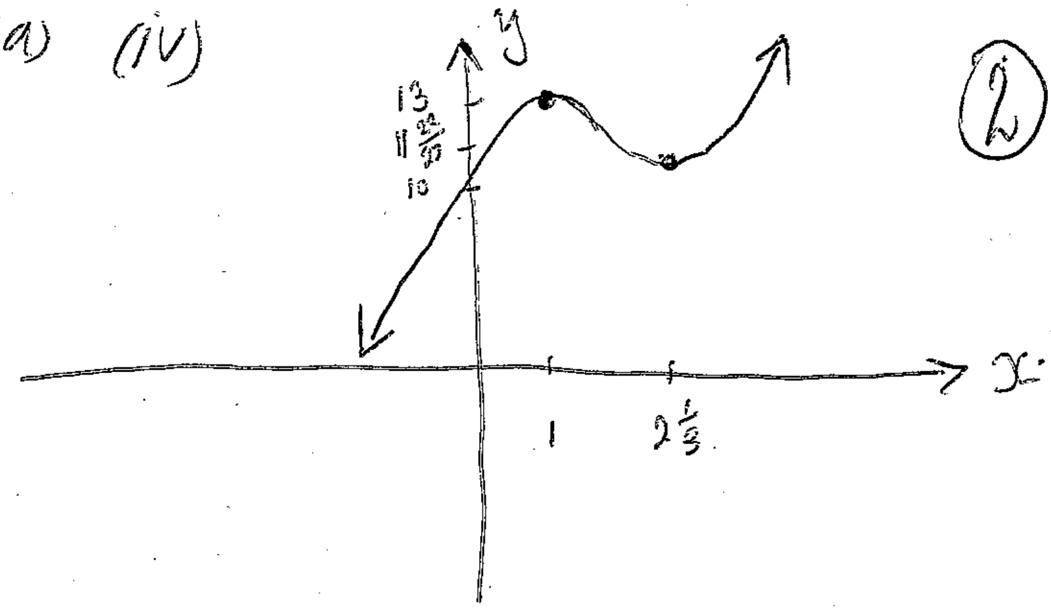
Using $(y - y_1) = m(x - x_1)$

$(y - 10) = 7(x - 0)$

$y - 10 = 7x$

$y = 7x + 10$ or $7x - y + 10 = 0$ 1

3 (a) (iv)



2

3 (b) Borrowed \$40,000 1.5% m. reducible

(i) equal monthly instalments $6\times12=72$

M = monthly instalments

$\$A_n$ = value of loan

$\$A_1 = 40{,}000 + 40{,}000\times1.5\% - m$

$40000\left(1+\frac{1.5}{100}\right)-m \quad = 40000(1.015)^1-m$

$\$A_2 = 40{,}000(1.015)^2 - m(1+1.015^1)$

$\therefore \$A_{72} = 40{,}000(1.015)^{72} - m(1+1.015^1+\dots 1.015^{71})$

$$m = \frac{40000(1.015)^{72}}{(1+1.015+\dots+1.015^{71})}$$

$$m = \frac{40{,}000(1.015)^{72}}{\frac{1(1.015^{72}-1)}{1.015-1}} = \frac{116846.3184\times0.015}{1.921157961}$$

$m = \$912.31$ 3

(ii) $\$912.31\times72 - \$40{,}000 = \$25686.32$

So $25686.32 = 40\cancel{000}\times\frac{R}{\cancel{100}}\times6$

$R = 10.7\%$ pa to 1DP 2

3 (c) $1x^2 - px + q = 0$

$\alpha + \beta = -\frac{b}{a} = p$ 1

$\alpha\beta = \frac{c}{a} = q$ 1

$y^2 - (\alpha+\beta+1)y + (\alpha^2+\beta^2) = 0$

$y^2 - (\alpha+\beta+1)y + (\alpha+\beta)^2 - 2\alpha\beta = 0$

$y^2 - (p+1)y + p^2 - 2q = 0$ 1

equal roots $\Rightarrow$ $b^2 - 4ac = 0$

$(p+1)^2 - 4 \times 1 \times (p^2 - 2q) = 0$

$p^2 + 2p + 1 - 4p^2 + 8q = 0$

$-3p^2 + 2p + 1 + 8q = 0$

$8q = 3p^2 - 2p - 1$

$q = \frac{1}{8}(3p^2 - 2p - 1)$ 1

2 UNIT Yr 11

Q 4.

(a) $f(x) = 2\sqrt{x-1} + 3$

Domain $x - 1 \geq 0$ Range $y \geq 3$ ✓ 2

$x \geq 1$ ✓

(b) $S_n = n^2 - 3^n$

(i) $S_{12} = 12^2 - 3^{12}$

$= -531297$ ✓ 1

(ii) $t_{12} = S_{12} - S_{11}$

$= -531297 - (11^2 - 3^{11})$ ✓

$= -531297 - (-177026)$ 2

$t_{12} = -354271$ ✓

(c) (i) P(Jerry wins) = P(Ben loses and Jerry wins)

$= \frac{\cancel{30}\,5}{36} \times \frac{\cancel{6}}{\cancel{36}\,6}$ 1

$= \frac{5}{36}$ ✓

(ii) P(Jerry wins on 1st or 2nd throw)

= P(Ben loses, Jerry wins) + P(B loses, J loses, B loses, J wins)

$= \frac{5}{36} + \frac{\cancel{30}\,5}{36} \times \frac{\cancel{30}\,5}{\cancel{36}\,6} \times \frac{\cancel{30}\,5}{\cancel{36}\,6} \times \frac{\cancel{6}}{\cancel{36}\,6}$ 1

$= \frac{5}{36} + \frac{5^3}{6^4}$. ✓

(iii) $P(\text{Jerry wins}) = \frac{5}{6^2} + \frac{5^3}{6^4} + \frac{5^5}{6^6} + \dots$ ✓

S_∞ where $a = \frac{5}{6^2}$, $r = \frac{5^2}{6^2}$ ✓ 3

$S_\infty = \frac{a}{1-r} = \frac{5}{6^2} \div \left(1 - \frac{5^2}{6^2}\right) = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$ ✓

4(d)(i) $y'(2) = 3x^2 - 2x + k = 0$

$\Rightarrow 12 - 4 + k = 0$

$\underline{k = -8}$

(2,0)

2

(ii) Eqn of curve, $y = \int (3x^2 - 2x + 8)\, dx$

$y = x^3 - x^2 - 8x + C$

2

$(2,0) \Rightarrow 0 = 8 - 4 - 16 + C$

$C = 12$

$\therefore y = \underline{x^3 - x^2 - 8x + 12}$.

Q5

(a) (i) Amt after 1 month

$= A_1 = 2500 \times 1.005$

$A_2 = 2500 \times 1.005^2 + 2500 \times 1.005$

$= 2500(1.005^2 + 1.005)$

$A_n = 2500(1.005^n + 1.005^{n-1} + \dots + 1.005)$

$= 2500\left(\dfrac{1.005(1.005^n - 1)}{1.005 - 1}\right)$

$= 502500(1.005^n - 1)$ 3

(ii) 31/12/30 ⇒ 192 months

$A_{192} = 502500(1.005^{192} - 1)$

$= 806741.9833$

$\approx \$806742$ 2

(iii) $600000 = 502500(1.005^n - 1)$

$1.005^n - 1 = \dfrac{6000}{5025}$

$1.005^n = \dfrac{6000}{5025} + 1$

$n \log 1.005 = \log\left(\dfrac{6000}{5025} + 1\right)$

$\therefore n = \dfrac{\log\left(\frac{6000}{5025} + 1\right)}{\log 1.005}$

$= 157.5405\dots$ 3

∴ Exceeds \$600000 in 158 months = 13 years 2 months

∴ 1st day of retirement = 1st March 2028

(iv) $600000 = \dfrac{M(1.005)(1.005^{132} - 1)}{0.005}$

$M = \dfrac{600000}{201(1.005^{132} - 1)}$

$= 3204.199777\dots$

$\approx \$3204.20$ 2

(b) (i) $V = \pi(3x)^2 h + 2 \times \frac{1}{3} \times \pi \times (3x)^2 \times 4x$

$V = 9\pi x^2 h + 24\pi x^3$

$9\pi x^2 h = V - 24\pi x^3$

$h = \dfrac{V - 24\pi x^3}{9\pi x^2}$ 2

(ii) $SA = 2\pi(3x)h + 2 \times \pi \cdot 3x \cdot 5x$

$= 6\pi x\left(\dfrac{V - 24\pi x^3}{9\pi x^2}\right) + 30\pi x^2$

$= \dfrac{2V - 48\pi x^3}{3x} + 30\pi x^2$

$= \dfrac{2V - 48\pi x^3 + 90\pi x^3}{3x}$

$= \dfrac{2V + 42\pi x^3}{3x}$

$\text{Cost} = \left(\dfrac{2V + 42\pi x^3}{3x}\right) \times \dfrac{3k}{2}$

$= \dfrac{2(V + 21\pi x^3)k}{2x}$

$= \dfrac{Vk + 21\pi x^3 k}{x}$

$= \dfrac{Vk}{x} + 21\pi x^2 k$ 3

(iii) $\dfrac{dC}{dx} = -Vkx^{-2} + 42\pi kx$

If $\dfrac{dC}{dx} = 0$: $\dfrac{Vk}{x^2} = 42\pi kx$

$Vk = 42\pi kx^3$

$\therefore x^3 = \dfrac{Vk}{42\pi k}$

$x^3 = \dfrac{V}{42\pi}$

$x = \sqrt[3]{\dfrac{V}{42\pi}}$

$\dfrac{d^2C}{dx^2} = 2Vkx^{-3} + 42\pi k$

$= \dfrac{2Vk}{x^3} + 42\pi k$

If $x = \sqrt[3]{\dfrac{V}{42\pi}}$ $\dfrac{d^2C}{dx^2} = \dfrac{2Vk}{\left(\frac{V}{42\pi}\right)} + 42\pi k$

$= 84\pi k + 42\pi k$

$= 126\pi k > 0$

∴ min value when $x = \sqrt[3]{\dfrac{V}{42\pi}}$ 3

(iv) $x : h = x : \dfrac{V - 24\pi x^3}{9\pi x^2}$

$$= 9\pi x^3 : (V - 24\pi x^3)$$

$$= 9\pi \cdot \frac{V}{42\pi} : \left(V - 24\pi \cdot \frac{V}{42\pi}\right)$$

$$= \frac{3V}{14} : V\left(1 - \frac{4}{7}\right)$$

$$= \frac{3V}{14} : \frac{3V}{7}$$

$$= \frac{3}{14} : \frac{3}{7}$$

$$= 1 : 2$$

2